JOINT LOCAL AND NON-LOCAL PRIORS FOR GROUND-BASED ASTRONOMICAL IMAGE DENOISING

Luxin Yan, Wenshan Liao, Yi Chang and Chunan Luo

{yanluxin, liaowenshan, yichang, chunanluo}@hust.edu.cn
School of Automation, Huazhong University of Science and Technology, China

ABSTRACT

Ground-based astronomical images usually suffer from the degradation of random noise, which has a negative impact on the subsequent performance. It is expected that local and nonlocal variations are the two sides of the same coin in image modeling. However, most of existing denoising methods focus on modeling the sparsity of local patches or the non-local self-similarity information individually, which may result in poor image structure representation ability. To remedy this issue, we propose a unified approach via combining the framelet and the low-rank prior scheme for astronomical image denoising. On one hand, the synthesis-based low-rank prior is employed to reveal the intrinsic low-dimensional subspace (namely the main edges) of the similar patches. On the other hand, the analysis-based framelet prior is introduced to capture the local subtle texture structures. Experimental results validate the effectiveness of this combination, and the proposed method outperforms the state-of-the-art denoising methods.

Index Terms— astronomical images, denoising, low-rank, framelet.

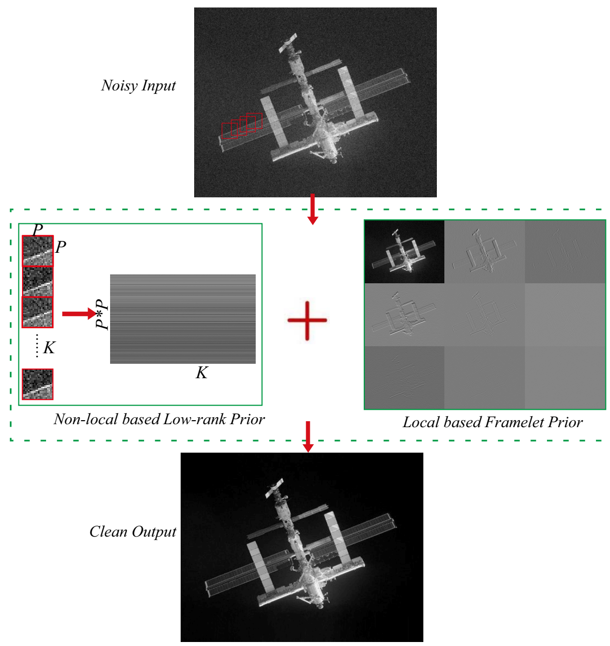

Fig. 1. Flowchart of the proposed denoising algorithm.

1. INTRODUCTION

Astronomical images are always contaminated by random noise due to long exposure times and photon noise. With the rapid development of the space program, the noise removal of the space object images taken by earth-based telescopes has become a necessary procedure. Over the past decades, many general and astronomical image specific noise reduction methods have been proposed [1, 2, 3, 4, 5, 6, 7, 8]. The astronomical image denoising methods can be classified into different categories. In this work, we classify the astronomical image denoising methods from local-based [1, 2, 3, 4, 5] and non-local-based perspective [6, 7, 8].

The local-based methods mainly focus on pursuing the sparsity in various transformed domains of the local image patches. The widely used wavelet transform, which benefits us from its multiscale analysis ability, has been well-adapted to the study of astronomical image denoising [9]. In [2], Donoho proposed to adaptively suppress the noise via soft-thresholding in the wavelet domain. The total variational [1] has also been used to remove the noise in astronomical image, due to its desirable properties such as convexity and the ability to preserve sharp edges. In [4], the authors proposed a TV-based astronomical image denoising method, assuming that local neighboring pixels should have similar values. In [5], the dictionary learning methods has also been introduced to remove the random noise in astronomical image.

The non-local-based methods make use of the pattern recurrence property in images. The non-local based method BM3D [6] is well-known due to its satisfactory performance and high efficiency for various noisy images. In [7], the K-SVD method has been extended to non-local case naturally. More recently, the work in [8] incorporates the weighted low-rank constraint for general image denoising, and have

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achieved the state-of-the-art performance.

In this work, we propose to jointly utilize local and non-local information for astronomical images denoising (Figure 1). The local-based framelet [10] with multiresolution analysis is introduced to achieve a more representative ability for the image textures. From non-local perspective, the low-rank is involved to model the similar patterns (mainly for the principle edges), in which they lie on a low-dimensional latent subspace. These two priors are essentially complementary to the other one, which provides a more complete representation for the images. Consequently, the fine textures and also the large scale edges can be well preserved in the restoration. The experiments demonstrate that our approach can obtain better denoising performance compared with previous methods.

2. PROPOSED ALGORITHM

Based on the additive Gaussian noise assumption, the image degradation model can be mathematically formulated as: $\boldsymbol{g} = \boldsymbol{f} + \boldsymbol{n}$, where $\boldsymbol{g} \in \mathbb{R}^{MN\times 1}$ is the the observed noisy image, $\boldsymbol{f}$ and $\boldsymbol{n}$ represents the ground truth image and noise, M and N denotes the number of the rows and columns, respectively. Estimating $\boldsymbol{f}$ from $\boldsymbol{g}$ can be possible if we have enough prior knowledge about the sparsity of the astronomical image in transformed domains.

In this work, we enforce two constraints on the image from both the local and non-local viewpoint. As shown in Fig. 1, the local structures, such as aerials and planes, exhibit strong directions and multiscale characteristics. Essentially, framelet could provide efficient representations to model the multidirectional and multiscale properties. With this constraint, the proposed method could restore the image with fine details. The low-rank prior can capture the underlying structure of the non-local similar patches. For the astronomical images including many man-made objects, there exist significant strong edge pattern, which inevitably facilitates the similar patches searching and boosts the final restoration result. By explicitly incorporating framelet and low-rank priors into a unified framework with appropriate regularization parameters, the final problem can be formulated as the following unconstrained optimization problem:

$$\left\{\hat{\boldsymbol{f}}, \hat{\boldsymbol{L}}_i\right\} = \arg\min_{\boldsymbol{f},\boldsymbol{L}_i} \|\boldsymbol{g} - \boldsymbol{f}\|_2^2 + \beta\|\boldsymbol{W}\boldsymbol{f}\|_1 + \eta \sum_i \left(\left\|\tilde{\boldsymbol{R}}_i\boldsymbol{f} - \boldsymbol{L}_i\right\|_F^2 + \lambda\|\boldsymbol{L}_i\|_*\right), \tag{1}$$

where $\boldsymbol{W}$ represents the framelet transform using the filters of the framelet system [1], $\tilde{\boldsymbol{R}}_i \in \mathbb{R}^{mn\times MN}$ is the patch extraction matrix operator, m and n is the row and column of extracted patch, $\boldsymbol{L}_i$ is the desired clean low-rank matrix. $||\bullet||_*$ denotes the nuclear norm [11], β, η, and λ are the corresponding regularization parameters. Our final model (1) is simple and easy to understand. The first term is the constraint of linear measurement. The second and the third terms are the framelet and low-rank prior enforced to preserve both the fine details and edge structures. The proposed method can exploit simultaneously the local and non-local information, which facilitate the final denoising result.

[1] In this work, we use the B-splines framelet [10].

2.1. Optimization

The proposed objective functional (1) can be efficiently solved by alternatively minimizing strategy with respect to the whole image $\boldsymbol{f}$ and low-rank matrix $\boldsymbol{L}_i$ at per each location, so as to split the original problem into two simpler subproblems as follows:

$\boldsymbol{L}_i$-subproblem: Omitting the terms independent of $\boldsymbol{L}_i$ in (1), we obtain following subproblem:

$$\hat{\boldsymbol{L}}_i = \arg\min_{\boldsymbol{L}_i} ||\tilde{\boldsymbol{R}}_i\boldsymbol{f} - \boldsymbol{L}_i||_F^2 + \alpha||\boldsymbol{L}_i||_*. \tag{2}$$

Equation (2) is the typical low-rank matrix approximation problem which has a closed-form solution and can be easily solved by the singular values thresholding algorithm [12]. In our implementation, we borrow the idea of the reweighting strategy to improve the performance.

$\boldsymbol{f}$-subproblem: After solving for $\boldsymbol{L}_i$, the latent image $\boldsymbol{f}$ can be reconstructed by solving optimization problem:

$$\hat{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}} ||\boldsymbol{f} - \boldsymbol{g}||_2^2 + \tau \sum_i ||\tilde{\boldsymbol{R}}_i\boldsymbol{f} - \boldsymbol{L}_i||_F^2 + \beta\|\boldsymbol{W}\boldsymbol{f}\|_1. \tag{3}$$

The difficulties in determining $\boldsymbol{f}$ are that the framelet-related l_1-norm term is nonsmooth and nonseparable. We introduce the alternating direction methods of multipliers [13] to solve it. The algorithm procedure of the proposed method is summarized in **Algorithm 1**.

Algorithm 1 **F**ramelet-regularized **L**ow-**R**ank (**FLR**)

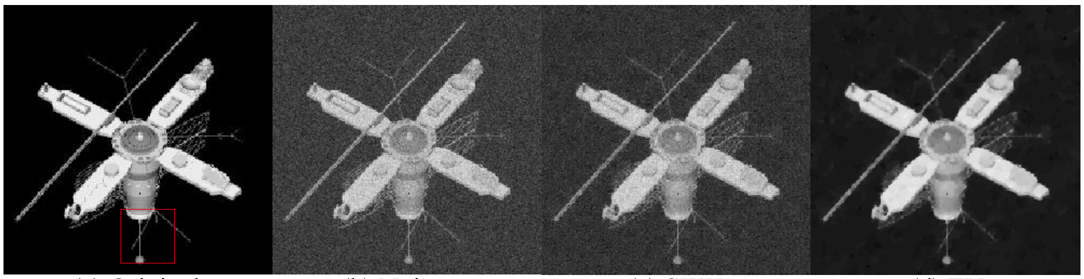
Require: Degraded image $\boldsymbol{g}$
1: **Initialize:**
2: • Set parameters β, η, and λ;
3: • Initialize $\boldsymbol{f}^{(1)} = \boldsymbol{g}$;
4: **for** n=1:N **do**
5: Compute $\boldsymbol{L}_i$ by solving Eq. (2);
6: Solve Eq. (3) for $\boldsymbol{f}^{n+1}$;
7: **end for**
Ensure: Clean Image $\boldsymbol{f}$.

3. EXPERIMENTAL RESULTS

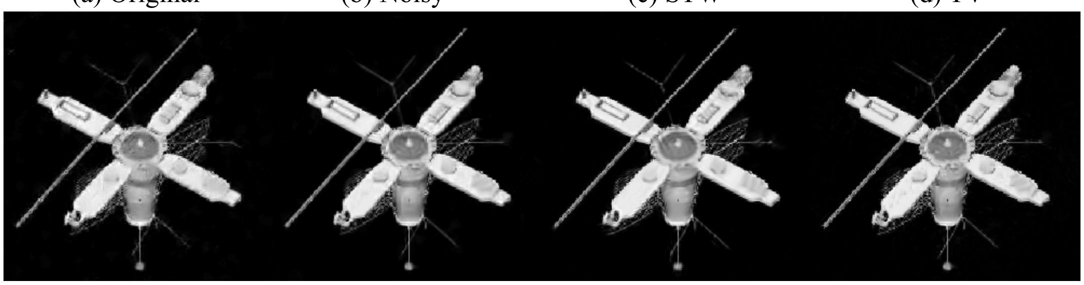
In the experiments, we choose several representative local or non-local based methods for comparison: TV [1], SHW [2], K-SVD [5], BM3D [6], and WNMM [8]. The first three methods are local-based, while the last two are non-local based. All the parameters are fine-tuned by default or following the

Table 1. Quantitative results of different methods under several noise levels on *satellite*.

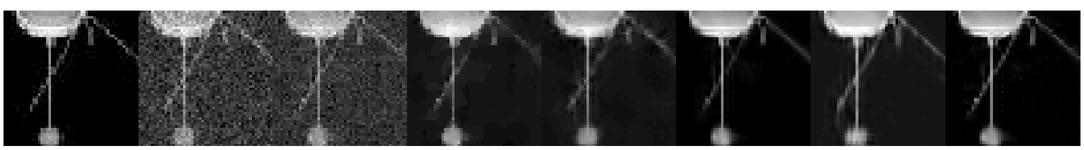
Sigma	Index	Methods						
		Noisy	STW	TV	KSVD	BM3D	WNNM	FLR
10	PSNR	28.1319	29.4393	31.2411	34.3508	35.0565	35.0764	**35.3709**
	SSIM	0.5114	0.7084	0.9195	0.9043	0.9464	0.9528	**0.9541**
30	PSNR	18.5895	21.9756	27.0071	27.5166	28.0571	28.3862	**28.7023**
	SSIM	0.2354	0.2993	0.7094	0.6858	0.8191	**0.8619**	0.8604
50	PSNR	14.1525	16.6895	24.0043	24.8813	24.7775	25.7101	**25.8836**
	SSIM	0.1569	0.1861	0.6723	0.5317	0.6980	**0.7901**	0.7576



(a) Original (b) Noisy (c) STW (d) TV



(e) KSVD (f) BM3D (g) WNNM (h) FLR



Zoom Results

Fig. 2. Simulated random noise removal results of *satellite* under noise level $\sigma = 30$.

rules in their papers to achieve the best performance.[2] The Matlab code of proposed method can be downloaded at the authors homepage.[3]

In simulation experiment, the noisy image is synthesized in accordance with the degradation model. We performed the simulated experiment with three noise levels: 10, 30 and 50. In Fig. 2, we show the denoising results under $\sigma = 30$ as an example. We observe that the result of the proposed method (the zoom region) has more clear textures and fewer artifact, which validates the effectiveness of the framelet prior. In Table 1, we show the quantitative assessment comparison results (including PSNR and SSIM) under the three noisy levels. The best results are highlighted in bold. The proposed method mostly obtain higher PSNR and SSIM values over the other state-of-the-art methods. In real experiment, the denoising results[4] of all competing methods are shown in Fig. 3. The noise is significantly suppressed and the detail information is better preserved by the proposed method than others.

4. CONCLUSION

In this paper, we propose to jointly utilize the local and non-local information for astronomical image denoising. The framelet is introduced to model the local fine-grained textures, while the low-rank constraint on the non-local patches can reveal faithful subspace of the similar patterns. The local and non-local are complementary to the each other in image

[2]We downloaded all the codes from the authors' homepage.

[3]http://www.escience.cn/people/changyi/index.html

[4]Original data was downloaded from http://www.clarkvision.com/

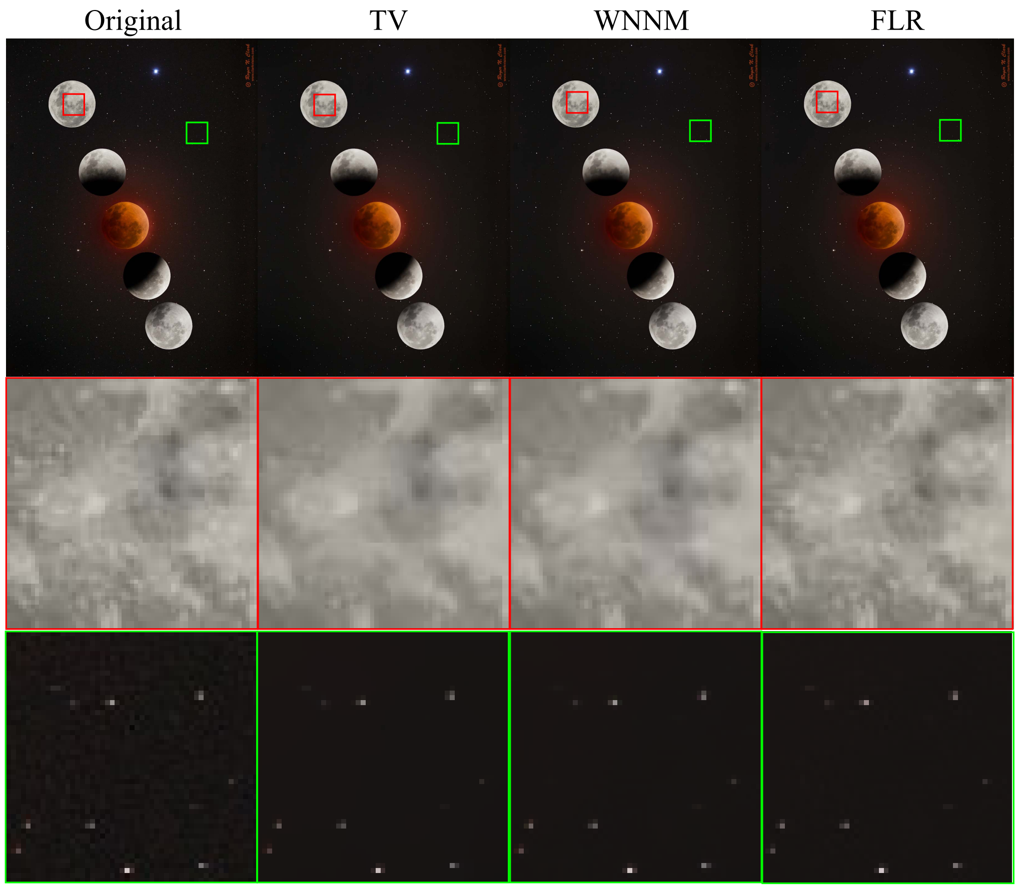


Fig. 3. Denoising results of a real astronomical image. The first to the fourth column show the noisy, TV, WNNM, and proposed FLR image, respectively. The first to the third row show original and zoom results.

structure representation. Within the unified denoising model, our approach can remove the random noise more thoroughly with better structure preserving ability. In the future, we will extend this work to other applications such as image deblurring, inpainting and so on.

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